Magnetic islands in a magnetized plasma with electron flow

J. Vranješ*
Institute of Physics, P.O. Box 57, Yu-11001 Belgrade, Yugoslavia
(Received 6 January 1998)

A system of two coupled nonlinear equations describing magnetic electron modes in a magnetized inhomogeneous plasma with a spatially dependent electron flow is derived. For a homogeneous basic state electron concentration, the two equations can be decoupled, and a nonlinear solution for the magnetic field in the form of a traveling stationary vortex chain of magnetic islands is found. [S1063-651X(98)13706-6]

PACS number(s): 52.35.Hr, 52.35.Mw

I. INTRODUCTION

Different forms of coherent structures, double and monopole vortices [1], and vortex chains [2,3], resulting from the self-organization of fusion and space plasmas, have attracted a lot of interest in the past 20 years. These coherent structures can appear in various processes such as nonlinear interaction of a strong pump propagating through a plasma in the processes of plasma heating, with slow low frequency perturbations normally existing in plasmas [4], in the development of different types of instabilities [5], etc. Since they can carry plasma particles effectively, investigations of vortices may be of great importance in problems of particle transport in fusion plasmas. A significant transport of particles appears in processes of inelastic collisions of vortices; in such situations there appear strong gradients of the electric field which, similar to the classic diffusion theory, cause particle transport perpendicularly to the magnetic field lines [6].

It is known that in plasmas with density and/or temperature gradients, magnetic electron modes can exist, localized to within a collisionless skin depth. They result from the self-generation of a localized magnetic field [7–9], and are of special importance in laser-fusion plasmas. Nonlinear equations describing perturbations of the electron component of an unmagnetized weakly nonuniform plasma have monopolar and dipolar moving vortex solutions [10], arising due to the dominance of vector-product type nonlinearities. Their group velocity, perpendicular to the density gradient in the basic state, is found to be larger than the velocity of corresponding linear waves.

In our earlier paper [11], we derived equations describing the nonlinear magnetic electron mode in a nonuniform (along the x axis), unmagnetized plasma, with a sheared plasma flow in the basic state. Such a flow introduces new terms in the corresponding evolution equations, which are responsible for the self-generation of coherent stationary nonlinear structures in the form of chains of magnetic islands. In the linear limit the general solution can be written as a combination of associated Legendre functions of degree 1, and of the order $\mu=(k_y^2+1)^{1/2}/\kappa$, where k_y is the wave number along the y axis, and κ is the characteristic length for the shear flow. The only linear solution localized along the x axis is obtained provided that the wave number k_y satisfies the condition $(k_y^2+1)/\kappa^2=1$. An instability of the electron magnetic mode, arising from the Cherenkov interaction with the inhomogeneous flow exists if $k_y>(\kappa^2-1)^{1/2}$, while in the opposite case the linear mode is damped. In the nonlinear regime, and for some new values of the parameters k_y and κ, two kinds of stationary solutions, in the form of moving single and double chains of magnetic field, localized in the direction of the shear flow gradient, and periodic along the flow, were found. An interesting feature of these nonlinear solutions is that the parameters k_y and κ have values for which the linear mode is highly unstable.

In this paper, we study nonlinear electron magnetic modes in a magnetized fusion plasma. The fact that plasma is magnetized in the basic state will give some extra terms in the corresponding equations, compared to basic equations in Ref. [11]. The plasma model is as follows: the spatially nonuniform magnetic field $\vec{B}_0=B_0(x)\vec{e}_z$ and the plasma concentration $n_0(x)$, causing an electron flow $v_0(x)\vec{e}_y$ in the basic state, perpendicular both to the magnetic field lines and the basic state gradients. The flow of this type is responsible for the creation of coherent stationary solutions in the form of magnetic chains. The approximation of immobile ions forming the neutralizing background of a plasma will be used, i.e., $\omega_{\rm pi}\ll\partial/\partial t$, where $\omega_{\rm pi}$ is the ion plasma frequency. Two coupled nonlinear equations for the perturbed magnetic field and the plasma temperature will be derived. In the linear regime, such equations belong to the class of equations describing a Kelvin-Helmholtz-type instability. Looking for stationary solutions, periodic along the y axis with the wave number k_y, traveling with a constant velocity u in the direction of the basic state electron flow, similar to Ref. [11], our nonlinear equations will be integrated once. Then, using the same procedure, they will be solved numerically. As in our Ref. [11], we shall attempt to find a class of solutions for corresponding plasma parameters.

II. BASIC EQUATIONS AND SOLUTIONS

We use the standard set of equations describing electron motion, i.e., the momentum equation, energy equation, and Maxwell equations:

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\vec{v}=-\frac{e}{m}(\vec{E}+\vec{v}\times\vec{B})-\frac{1}{mn}\vec{\nabla}(nT), \qquad (1)$$

*Electronic address: vranjes@phy.bg.ac.yu

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)(Tn^{1-\gamma})=0, \tag{2}$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{3}$$

$$\vec{\nabla}\times\vec{B}=-\mu_0 en\vec{v}. \tag{4}$$

Equations (1)–(4) describe electron perturbations in the limit of immobile ions, i.e., $\omega_{\rm pi}\ll\partial/\partial t\ll\omega_{\rm pe}$, $c\vec{\nabla}$. Thus we study slow electron motion, neglecting the displacement current in Eq. (4), and the density perturbations so that $n\equiv n_0(x)$. These assumptions are standardly used in the theory of generation of magnetic fields and magnetic vortices [8–11].

Stationary basic state, with the magnetic field given by $\vec{B}_0=B_0(x)\vec{e}_z$, is described by the following equation:

$$\frac{d}{dx}\left(\frac{B_0^2}{2\mu_0}+n_0T_0\right)=0. \tag{5}$$

Also, from Eq. (4), we have an expression for the basic state electron velocity:

$$\vec{v}_0(x)=-\frac{1}{\mu_0 en_0}\vec{\nabla}\times\vec{B}_0=\frac{c^2}{\omega_{pe}^2}\vec{e}_z\times\vec{\nabla}\Omega_0. \tag{6}$$

Here Ω_0 and $\omega_{\rm pe}$ are the electron gyrofrequency, and plasma frequency, respectively.

We study a two-dimensional motion of electrons, i.e., we seek z-independent solutions of the above equations. Taking the curl of Eqs. (1), using Eqs. (3)–(6), after some lengthy but straightforward algebra we have an equations for the perturbed magnetic field B_1:

$$\frac{\partial}{\partial t}\left(\frac{1}{\mu_0 en_0}\vec{\nabla}^2-\frac{e}{m}-\frac{n_0'}{\mu_0 en_0^2}\frac{\partial}{\partial x}\right)B_1 +\left[\frac{1}{\mu_0 m}\left(\frac{B_0}{n_0}\right)'-\frac{ev_0}{m}-\frac{1}{\mu_0 en_0}\left(v_0''-\frac{n_0'v_0'}{n_0}\right)\right]\frac{\partial B_1}{\partial y} -\frac{n_0'}{mn_0}\frac{\partial T_1}{\partial y}+\frac{v_0}{\mu_0 en_0}\frac{\partial}{\partial y}\vec{\nabla}^2B_1-\frac{v_0n_0'}{\mu_0 en_0^2}\frac{\partial^2B_1}{\partial x\partial y} +\frac{1}{(\mu_0 en_0)^2}\{B_1,\vec{\nabla}^2B_1\}=0. \tag{7}$$

In the same manner, we obtain an equation for the temperature:

$$\frac{\partial T_1}{\partial t}+\frac{1}{\mu_0 en_0}\{B_1,T_0+T_1\}+v_0(x)\frac{\partial T_1}{\partial y} +\frac{(\gamma-1)T_0}{\mu_0 en_0^2}\{n_0,B_1\}=0. \tag{8}$$

Here we use the Poisson bracket notation

$$\{B_1,\vec{\nabla}^2B_1\}\equiv\left(\frac{\partial B_1}{\partial x}\frac{\partial}{\partial y}-\frac{\partial B_1}{\partial y}\frac{\partial}{\partial x}\right)\vec{\nabla}^2B_1. \tag{9}$$

In the derivation of the above set of two coupled nonlinear equations, the second order scalar nonlinear term of the form $\{n_0,B^2\}$ has been neglected compared to the vector-product-type nonlinear term $\{B,\vec{\nabla}^2B\}$, as being much smaller in the regime when $\omega_{\rm pe}^2/k^2c^2$ is close to, or not much larger than, 1. It is assumed also that $|k^{-1}|\ll L_n$, L_T, where L_n and L_T are the characteristic lengths of inhomogeneity for n_0 and T_0, respectively.

In the local approximation along x, from Eqs. (7) and (8) we obtain the same dispersion equation as in Ref. [8],

$$(\omega-v_0k_y)^2+\alpha(\omega-v_0k_y)+\beta=0, \tag{10}$$

where

$$\alpha=-\frac{\lambda_S^2k_y}{\lambda_S^2k_y^2+1}\left(\lambda_S^2\Omega_0'''-\frac{\lambda_S^2\Omega_0''}{L_n}-\Omega_0'+\frac{\Omega_0}{L_n}\right),$$

$$\beta=\frac{k_y^2\lambda_S^2v_T^2}{\lambda_S^2k_y^2+1}\left(\frac{1}{L_nL_T}-\frac{\gamma-1}{L_n^2}\right),\quad \lambda_S=\frac{c}{\omega_{pe}},\quad v_T^2=\frac{T_0}{m}.$$

Obviously an oscillatory instability is possible if $\alpha^2/4<\beta$ and $L_nL_T>0$ (in the case of unmagnetized plasma, it is a purely growing one). Accordingly, perturbation (creation in the unmagnetized plasma case) of the magnetic field is closely connected with the direction of the basic state gradients n_0' and T_0'. It can be shown [8] in the same local approximation that, in the strongly nonlinear limit, Eqs. (7) and (8) possess stationary coherent solutions in the form of double vortex traveling with a constant velocity in the direction perpendicular to the basic state gradients, and these nonlinear solutions may represent the final stage of the above gradient driven instability.

In the nonlocal treatment, Eqs. (7) and (8) yield a complicated linear eigenvalue equation in the x direction, which is difficult to solve in general. The problem becomes much simpler when the effects of the concentration inhomogeneity are neglected. As follows from Eq. (10), the oscillatory instability driven by the density and temperature gradients in this case disappears, and Eq. (7) is decoupled from Eq. (8). Using the normalization

$$\lambda_S\vec{\nabla}\to\vec{\nabla},\quad \frac{eB_{0,1}}{m}\Big/\frac{\widehat{v_0}}{\lambda_S}\to\Omega_{0,1},\quad \frac{\lambda_S}{\widehat{v_0}}\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}, \tag{11}$$

with the basic state electron velocity written in the form $v_0(x)=\widehat{v_0}f(x)$, Eqs. (7) and (8) can be rewritten as

$$\left(\frac{\partial}{\partial t}+f\frac{\partial}{\partial y}+\vec{e}_z\times\vec{\nabla}\Omega_1\cdot\vec{\nabla}\right)(\vec{\nabla}^2-1)\Omega_1+(f-f'')\frac{\partial\Omega_1}{\partial y}=0, \tag{12}$$

$$\left[\frac{\partial}{\partial t}+\vec{e}_z\times\vec{\nabla}\Omega\cdot\vec{\nabla}\right]T=0. \tag{13}$$

In Eq. (13), we use notations $\Omega=\Omega_0+\Omega_1$ and $T=T_0+T_1$.

In the linear limit, for the perturbation of Ω_1 in the form $\hat{\Omega}_1(x)\exp(-i\omega t+iky)$, from Eq. (12) we have

$$\frac{d^2\hat{\Omega}_1(x)}{dx^2}+F(x)\hat{\Omega}_1(x)=0, \tag{14}$$

where

$$F(x)=-k^2-1+\frac{f(x)-f''(x)}{kf(x)-\omega}k. \tag{15}$$

Equation (14) belongs to the class of equations describing streaming instabilities, and in principle can be solved for some specific profiles of the flow. Thus, neglecting the density gradient, the system becomes subject to a completely different regime of instability. The possibility for instability can be demonstrated using the Rayleigh theory developed in the hydrodynamics. We multiply Eq. (14) by a complex conjugate $\hat{\Omega}_1^*(x)$, and integrate across the flow,

$$\int\left[\hat{\Omega}_1^*(x)\frac{d^2\hat{\Omega}_1(x)}{dx^2}+F(x)\hat{\Omega}_1^*(x)\hat{\Omega}_1(x)\right]dx=0, \tag{16}$$

where the frequency is complex in the general case, i.e., $\omega=\omega_r+i\ \omega_i$. Dividing the real and imaginary part of the above integral, and equating the last one with zero for $\omega_i\neq 0$, one can find that the instability is possible for the flow satisfying the following condition at any position along the x axis:

$$f''(x)-f(x)=0. \tag{17}$$

Thus, in the nonlocal treatment and in the absence of density gradients, a streaming type instability is possible, which in the strongly nonlinear limit we expect would eventually saturate into a type of stationary coherent solution in the form of a vortex chain traveling along the flow and localized across it. A similar situation was observed in the case of an unmagnetized plasma [11]. Consequently we shall concentrate on solving the nonlinear equation (12) in the strongly nonlinear regime, in order to show that such solutions exist.

We write $\partial/\partial t=-u\partial/\partial y$ using Eq. (6), and in this case Eq. (12) can be integrated once, giving

$$(\vec{\nabla}^2-1)\Omega=g(\Omega-ux). \tag{18}$$

A similar procedure to Eq. (13) yields

$$T=G(\Omega-ux). \tag{19}$$

Here $\Omega=\Omega_1+\Omega_0$ is the total (perturbed plus basic) magnetic field, and g and G are arbitrary functions of the same argument—stream function $\Omega-ux$. They will be chosen uniquely in such a way that Eqs. (18) and (19) are satisfied asymptotically for arbitrary solutions vanishing at $x\rightarrow\pm\infty$, i.e., in the region of open stream lines. In those regions of space where the stream lines are closed, the functions g and G in principle may have different forms, and, in order to work correctly, in addition to the boundary conditions at infinity it would be necessary to find the appropriate form of the functions in these regions, and to match solutions at separatrices. It would be a difficult task to work in that way and we shall take corresponding asymptotic expressions of the above functions to be valid everywhere. A detailed discussion concerning the choice and nature of such functions is given in Ref. [12]. Thus we choose the function g in the form

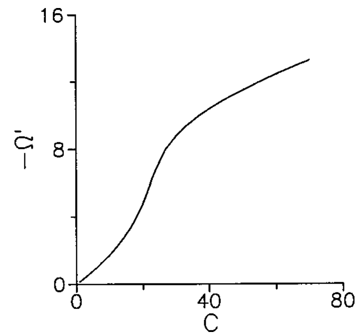

FIG. 1. Locus of the pairs $(-\Omega'(0),C)$ yielding localized and odd solutions of Eq. (22) along the x axis.

$$g(\Omega-ux)=C/[\exp(\Omega-ux)+\exp(-\Omega+ux)], \tag{20}$$

where C is an arbitrary constant. It will be varied in order to obtain localized solutions in the x direction. We look for the solution of Eq. (12) in the form

$$\Omega(x,y)=\Omega_x(x)+\delta\Omega_x(x)\cos(ky), \tag{21}$$

where $|\Omega_x|\gg|\delta\Omega_x|$. This yields the following two equations for Ω_x and $\delta\Omega_x$:

$$\left(\frac{\partial^2}{\partial x^2}-1\right)\Omega_x=\frac{C}{2\cosh(\Omega_x-ux)}, \tag{22}$$

$$\left(\frac{\partial^2}{\partial x^2}-k^2-1\right)\delta\Omega_x=-C\,\delta\Omega_x\frac{\sinh(\Omega_x-ux)}{2\cosh^2(\Omega_x-ux)}. \tag{23}$$

The above set of equations is solved numerically. The second order differential equation (22) is not coupled with Eq. (23), and we solve it from the point $x=0$, looking for a localized solution in the x direction. Changing the values of the derivative Ω_x' at $x=0$, and the constant C, we find a class of localized and odd solutions for certain pairs of C and $-\Omega_x'$, represented in Fig. 1. Equation (23) is linear, coupled with the former equation, and, according to expression (21), apart from the constant C it also depends on the parameter k. We look for its localized solution in the x direction. It turns out that all values of C are not allowed; well localized and even solutions of Eq. (23) are possible for the values of (C,k) given in Fig. 2. A typical appearance of the solutions of Eqs. (22) and (23) is given in Fig. 3. Here $C=20$, $k=0.759$, and $u=-4$. The difference between two neighboring magnetic field lines is $\Delta\Omega=0.2$.

It is interesting to note that Eq. (12) admits the steady state solutions [2], which can be found by using the following procedure. Put $\partial/\partial t=0$ in Eq. (12), i.e., $u=0$ in Eq. (18), and choose the arbitrary function g as

$$g(\Omega)=-\Omega+\frac{4Ak^2}{a^2}\exp\left(-\frac{2}{A}\Omega\right).$$

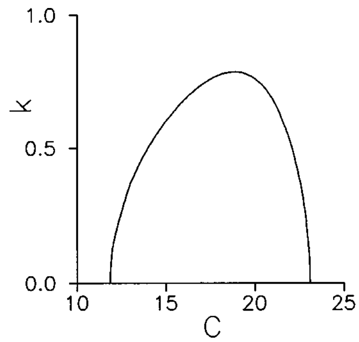

FIG. 2. Pairs of (C,k) giving well localized even solutions $\delta\Omega_x$ of Eq. (23).

In this case, Eq. (18) has a solution in the form

$$\Omega = A\ \ln\left[2\cosh(kx) + 2\sqrt{1-\frac{1}{a^2}}\cos(ky)\right]. \quad (24)$$

The above expression for $a^2 > 1$ represents a row of identical Stuart's vortices along the y axis. In the limit $a \to 1$, this gives the zonal flow. On the same condition, choosing g as

$$g(\Omega) = -\Omega - \frac{A}{4}(a^2 - b^2)\sinh\left(\frac{4\Omega}{A}\right),$$

where $b \leq a$, and A are arbitrary constants, one can find the solution of Eq. (18) in the form

$$\Omega = A\ \text{arctanh}\left(\frac{b\ \cos(ay)}{a\ \cosh(bx)}\right), \quad (25)$$

which represents a row of counter rotating vortices.

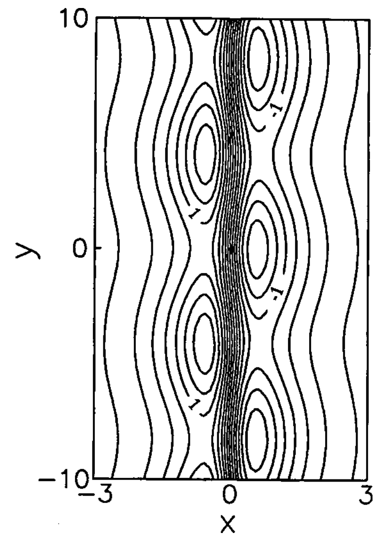

FIG. 3. Typical appearance for contour plots of the total magnetic field. Here the distance value of the magnetic field between two lines is $\Delta\Omega = 0.2$, and $C = 20$, $k = 0.759$, and $u = -4$. Quantities are dimensionless, normalized in accordance with expressions (11).

III. CONCLUSIONS

In this paper we have investigated the dynamics of strongly nonlinear electrons in a system with immobile ions of an inhomogeneous magnetized plasma with a spatially dependent magnetic field in the basic state. Such a spatial inhomogeneity of the magnetic field causes an electron flow in the basic state. In the linear limit, two types of instabilities are possible: in a local approach we have the gradient driven oscillatory instability, and in the nonlocal case the streaming instability. In the strongly nonlinear regime, the first instability may saturate into coherent structures in the form of double vortices. In the second case, neglecting density gradients, the corresponding nonlinear electron equations can be integrated once, and the pair of equations obtained in this way, describing total magnetic field and electron temperature, are solved numerically. We have found a range of corresponding parameters for which the solutions are localized, and are in the form of chains of islands.

The model used here can be of interest for an investigation of experimental plasmas and various magnetized plasma configurations in space, like the Earth magnetoplasma, or magnetic arcs on the Sun, in spite of the fact that the parameters in these systems differ very much. A good example of this is the analysis of data obtained recently from the Freja satellite [13]. Vortex solitons discovered in this way, with characteristic spatial scales of 300–600 m, can be nicely described using the standard nonlinear theory of drift waves, developed to describe drift wave turbulence in present day tokamak machines. Other examples are vortices and vortex chains obtained in the analysis of data from the satellite IC-B-1300 [14].

According to experimental and theoretical investigations of high-confinement modes in tokamaks the appearance of edge localized modes in experiments is now an established phenomenon. From this point of view, the formation of vortex chains in the edge region can act as a barrier to the particle transport. In the presence of finite dissipations, these structures have a finite lifetime. In some cases [15], because of the shear flow, they show an oscillatory behavior which is related to the transition from low to high regimes, and back (L-H-L transitions). These oscillations are closely connected with changes of the velocity shear; when shear becomes weak, fluctuations grow, and vice versa. In the case of chains associated with tearing modes [16], the adiabatic theory of their time evolution in the bulk plasma indicates an intermittent behavior, which may result in their destruction and stochastization of the magnetic field within a layer of the collisionless skin depth scale. In the same problem, another possibility predicts the stochastization on a longer time scale, through a sequence of bifurcations that corresponds to abrupt changes of vortex chain parameters like wavelength, speed, etc.

Recently, an interesting investigation of the influence of ion dynamics on generation of fluctuating magnetic field was performed [17], and some interesting types of linear instabilities on an ion time scale were found. This influence on the formation of coherent stationary chain structures would seem to be of interest to investigate, and this work is in progress.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science and Technology, Republic of Serbia. The author is grateful to Dr. D. Jovanović for helpful discussions.

[1] A. Hasegawa and K. Mima, Phys. Rev. Lett. **39**, 205 (1977); K. H. Spatschek *et al.*, *ibid.* **64**, 3027 (1990).
[2] P. K. Shukla and L. Stenflo, Astron. Astrophys. **300**, 933 (1995).
[3] J. Vranješ and D. Jovanović, Phys. Scr. **55**, 93 (1997).
[4] D. Jovanović and J. Vranješ, Phys. Rev. E **53**, 1051 (1996).
[5] P. K. Shukla and J. Weiland, Phys. Lett. A **136**, 59 (1989).
[6] W. Horton, Phys. Fluids B **1**, 524 (1989).
[7] R. D. Jones, Phys. Rev. Lett. **51**, 1269 (1983).
[8] L. Stenflo and M. Y. Yu, Phys. Fluids **29**, 2335 (1986); G. Murtaza, P. K. Shukla, M. Y. Yu, and L. Stenflo, J. Plasma Phys. **41**, 257 (1989).
[9] A. M. Mirza and P. K. Shukla, Phys. Plasmas **4**, 2302 (1997).
[10] M. Y. Yu, P. K. Shukla, and L. Stenflo, Phys. Fluids **30**, 3299 (1987); J. Nycander and V. P. Pavlenko, Phys. Fluids B **3**, 1386 (1991).
[11] J. Vranješ and D. Jovanović, Phys. Plasmas **3**, 2275 (1996).
[12] D. Jovanović and F. Pegoraro, Phys. Scr. (to be published).
[13] D. J. Wu, G. L. Huang, and D. Y. Wang, Phys. Rev. Lett. **77**, 4346 (1997); D. J. Wu, D. Y. Wang, and G. L. Huang, Phys. Plasmas **4**, 611 (1997).
[14] V. M. Chmyrev *et al.*, Planet. Space Sci. **39**, 1025 (1991); Phys. Scr. **38**, 841 (1988).
[15] A. Takayama, M. Wakatani, and H. Sugama, J. Phys. Soc. Jpn. **64**, 791 (1995).
[16] D. Jovanović and J. Vranješ, Phys. Plasmas **1**, 3239 (1994).
[17] H. Saleem, Phys. Rev. E **54**, 4469 (1996); A. M. Mirza and P. K. Shukla, Phys. Plasmas **4**, 2302 (1997); M. Khizar *et al.*, Phys. Scr. **55**, 599 (1997).